Using GIS and Modeling to Improve
Small Area School Enrollment Projections

By

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Abstract

The purpose of this paper is to develop a new approach using GIS and modeling to improve the accuracy of small area school enrollment projections, used as input to running regional transportation model for Southern California. Accurate school enrollment projections would help to increase the credibility of the regional transportation model results. The new approach utilizes a facility location model, land suitability analysis, a spatial interaction model to develop location of a new school and related school enrollment. GIS techniques including surface model and location model are used in the land suitability analysis. Our study indicates that modeling processes with reasonable assumptions (e.g., school capacity and a new school) can produce accurate and reasonable student enrollment projections of small areas. The new approach is expected to be used as a planning tool for environmental planners, land use planners, and school district demographers as well as transportation planners, because the new approach has strong planning implication for future accessibility, mobility, air quality, environment, and use of energy (e.g., gasoline).

1. Introduction

School enrollment[1] plays an important role in estimating school related trips in the transportation modeling process of the Southern California Association of Governments (SCAG). School related trips account for more than 10 percent of total person trips occurred during any single day (SCAG, 2003. p.33). The more accurate forecasts of school enrollments would result in better transportation model result.

The existing approach of developing school enrollments used by SCAG staff is based on the current relationship between student enrollments of existing schools at the Census Tract and Transportation Analysis Zone (CT-TAZ)[2] level and student enrollments of existing schools at the City level. TAZ data is sum of CT-TAZ data within the TAZ. The existing approach does not project accurate school enrollments, because it uses a constant ratio of CT-TAZ level enrollments to the City level enrollments, and it does not assume any possibility of expanding existing schools or locating new schools. The problem becomes severe in terms that newly developing CT-TAZs would not have any new schools and related school enrollments, which would be used as a basis for calculating the ratio of CT-TAZ level enrollments to City level enrollments.

The new approach is proposed to improve the accuracy of projecting the small area school enrollments. The new modeling approach, using a facility location model, a GIS-based suitability analysis, and a spatial interaction model is designed to develop school enrollment projections by using the proper linkage of population, accessibility, and other physical land characteristics. The new approach is expected to be used as a planning tool for environmental planners, land use planners, and school district demographers as well as transportation planners, because the new approach has strong implication for discussion and planning for future accessibility, mobility, air quality, environment, use of energy (e.g., gasoline).

2. Existing Approach

There are three frequently used methods of projecting school enrollments: 1) a cohort-progression method; 2) enrollment ratio method; 3) housing-unit method. (George, M.V., Stanley K. Smith, David A. Swanson, and Jeff Tayman, 2004.; Siegel, Jacob S., 2002.; Plane, David A. and Peter A. Rogerson, 1994.).

SCAG traditionally projects school enrollments using two types of school enrollments ratios; 1) ratio of the number of enrolled students (K-12 or college) to the number of total population for the city level, 2) a ratio of the number of K-12 enrolled students of age 5-17 to the number of children of age 5-17 for the county level or a ratio of the number of College enrolled students of age 18-24 to the number of persons of age 18-24 for the county level (See table 1). The city level enrollments use an overall enrollment ratio of total population, while the county level enrollments use an age-specific enrollment ratio.

Table 1. Existing School Enrollment Projection Methodology

Geography	K-12	College
County	Enrollment ratio of persons of age 5-17	Enrollment ratio of persons of age 18-24
City	Enrollment ratio of city population	Constant share of County college enrollments
CT-TAZ	Constant share of City K-12 enrollments	Constant share of City college enrollments

The county level enrollment projections are based on age-specific enrollment ratios and age-specific population projection. For example, assuming that a specific county indicates age-specific school enrollments at 95% of the number of population of age 5-17, school enrollment ratio of 95% is multiplied by projected population of age 5-17 to produce school enrollment projections.

The city level enrollment projections are based on overall enrollment ratios and total population projection. As an example, if a specific city’s overall school enrollments are estimated at 20% of the number of total population, school enrollment projections are derived by multiplying school enrollment ratio of 20% by projected total population.

The CT-TAZ level enrollment projections are based on the constant share method. CT-TAZ level school enrollment projections are computed by multiplying the base year CT-TAZ’s share of the city’s base year school enrollments by projected city level school enrollments.

The existing enrollment ratios method is generally found to be acceptable for county and city level enrollment projections. But the constant share method creates problems in projecting CT-TAZ level enrollment projections for several reasons. The first major issue is that the zonal-based aggregate school enrollment projections at the CT-TAZ level do not consider the linkage between population growth, accessibility to individual schools, possibility of expanding existing schools or building new schools. The lack of consideration of linkages between the above elements results in inaccurate projection of school enrollments. Second, more specifically, newly developing areas with no existing schools would not get school enrollment projections along with population projection. The reason is that there is no record of the relationship between population and school enrollments in the base year. Third, the constant share method does not consider land availability and suitability for locating schools. The unsuitability of land for future school locations will limit the expansion of existing schools and the construction of new schools.

3. New Approach

A new approach toward projecting school enrollments at CT-TAZ level is the individual school-based enrollment projection method. A new approach is a bottom-up approach reflecting the linkage between population growth, accessibility to individual schools, possibility of expanding existing schools or building new schools, and availability of suitable land for new schools. The new approach is based on four sequential modeling processes: 1) small area population projection modeling; 2) facility location modeling; 3) land suitability analysis; 4) spatial interaction modeling. Ottensmann (2000) describes how to use facility location modeling and spatial interaction modeling in a spreadsheet environment for the optimal public library location assignment.

1) Small area population projection modeling

As a first step of developing school enrollment projections, population projections should be made at CT-TAZ level. The size and age composition of population at CT-TAZ level would determine the future demand of expanding existing schools or constructing new schools.

The small area population projection is prepared using several methods. A widely used method is a comparative method based on both the past trends of small area population and the relationship between small area population and large area population (Davis, 1995). The small area population projection is also made using microgeographic land use and activity models. Two typical models include urban development model developed by San Diego Association of Governments and subarea allocation model developed by Maricopa Association of Governments. (Stanley K. Smith, Jeff Tayman, and David A. Swanson, 2001). The recently used small area population method is scenario planning (SCAG, 2004). Given the complexity of the issues we face in today's environment, the number of variables that have to be considered, and the planning horizon time frame, it is apparent that getting the right projection is not possible. It might be effective to project the future by putting forth possible future scenarios.

It is clear that diverse methods are utilized to reach the small area population. Our study focuses on three modeling processes: facility location modeling; land suitability analysis; and spatial interaction modeling, assuming that population projection is provided through a separate modeling process.

2) Facility location modeling

A facility location model is used to find preliminary optimal school locations by minimizing the total distances between population (demand points) and potential school sites (facility). The distance can be measured in a physical distance (for example, Euclidean distance, Manhattan distance, P root squared distance), a time distance, or a cost distance (Yoon & Yoon, 2004). Our study minimizes the distance from demand points to a facility by using a Euclidean distance[3]. Although a network based time distance might measure the distance more accurately, a Euclidean distance is frequently used for its simple computation process (Ottensmann, 2000). The formula to minimize the distance is as follows:

$$Minimize \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} w_i d_{ij} ,$$

Where $a_{ij} = 1$ when user from demand point i is assigned to facility j, and $a_{ij} = 0$ when otherwise. d_{ij} is the distance from i to j. w_i is the weight, usually measured in the number of users from demand point i. Since there are no direct solutions for determining the optimal locations, a process is iterated until an optimal solution is found.

3) Land suitability analysis

The third step is to search a suitable site for new schools using GIS. Land suitability analysis uses multiple sources of information related to hydrologic, geologic, and biologic features of a site, accessibility of a site (to infrastructure and urban land uses), socioeconomic features (Berke et al, 2005). The land suitability is being calculated by overlaying maps of various features. A range of popular methods include: 1) pass/fail screening (with a minimum acceptable rating), 2) equivalent rating (with a suitability value for each type of features for a particular land use), 3) weighted rating (with a weight to each feature), 4) direct assignment rating (based on a combined examination of data from all features).(Berke, 2005). A weighted rating method is used in our study due to its wide applications (Randolph, 2004; Allen, 2001; Klostermann, 2001). In a weighted rating method, the highest total score implies the most suitable for a specific land use. Total score is determined in a following way:

$$TotalScore = (Wa * Ra) + (Wb * Rb) + (Wc * Rc) \ldots.(Wn * Rn)$$

Where W = weight, R= rating, a,b,c,…n=features

4) Spatial Interaction Modeling

The fourth and last step is to apply a spatial interaction model, also known as a single-constrained gravity model to predict the size of new schools. This single-constrained gravity model has been widely used to project residential activities, retail trade volumes, and recreational facility utilization (Ottensmann, 1985). This model assumes that the new school enrollments are determined by three major components: the number of school-age children at the origin CT-TAZs; attractiveness of the potential school site at the destination CT-TAZs; and the distance between the origin CT-TAZs and the potential school site. The preliminary school site was determined using a facility location model, and was refined using land suitability analysis. The spatial interaction model will be used to determine the number of school enrollments with introduction of changing assumptions. A spatial interaction model is represented as follows (Ottensmann, 2000)

$$D_j = K \sum_i \left[\frac{O_i A_{ji} f(d_{ij})}{\sum_j A_j^a f(d_{ij})} \right]$$

Where $f(d_{ij}) = \frac{1}{d_{ij}^b}$

D_j is the total number of school enrollments in destination *school j*. O_i is the number of school age population at origin *CT-TAZ i*. A_j is the attractiveness measure for destination *school j*. The distance from origin *CT-TAZ i* to destination *school j* is d_{ij}. The distance function $f(d_{ij})$ is the probability of school commuting from origin *CT-TAZ i* to destination *school j*, reflecting a distance decay function. K is a balancing factor. a and b are empirically derived parameters. The distance function $f(d_{ij})$ above is based on a declining power function, but sometimes is based on a negative exponential function, a modified lognormal function, and a modified gamma function (Smith et al, 2001). The model can be easily run using spreadsheets (e.g., MS Excel) due to increased capacity of personal computers (Ottensmann, 2000).

4. Case Study: The City of Palmdale, California

The City of Palmdale, incorporated in 1962, is located in the Northern Part of Los Angeles County (See figure 1). As one of the fastest growing cities in America, Palmdale's population has increased from 12,227 in 1980 to 116,670 in 2000. The City is expected to continue growing fast in the future. The city will add approximately 220,000 persons between 2000 and 2030 in the 2004 Regional Transportation Plan (RTP) by the SCAG (http://scag.ca.gov/forecast/index.htm). The growth rate of 190% during the projection period of 2000 and 2030 is the highest among cities in the Los Angeles County. Given the fast population growth, Palmdale would be a good candidate for assessing new modeling techniques and GIS for projecting school enrollments.

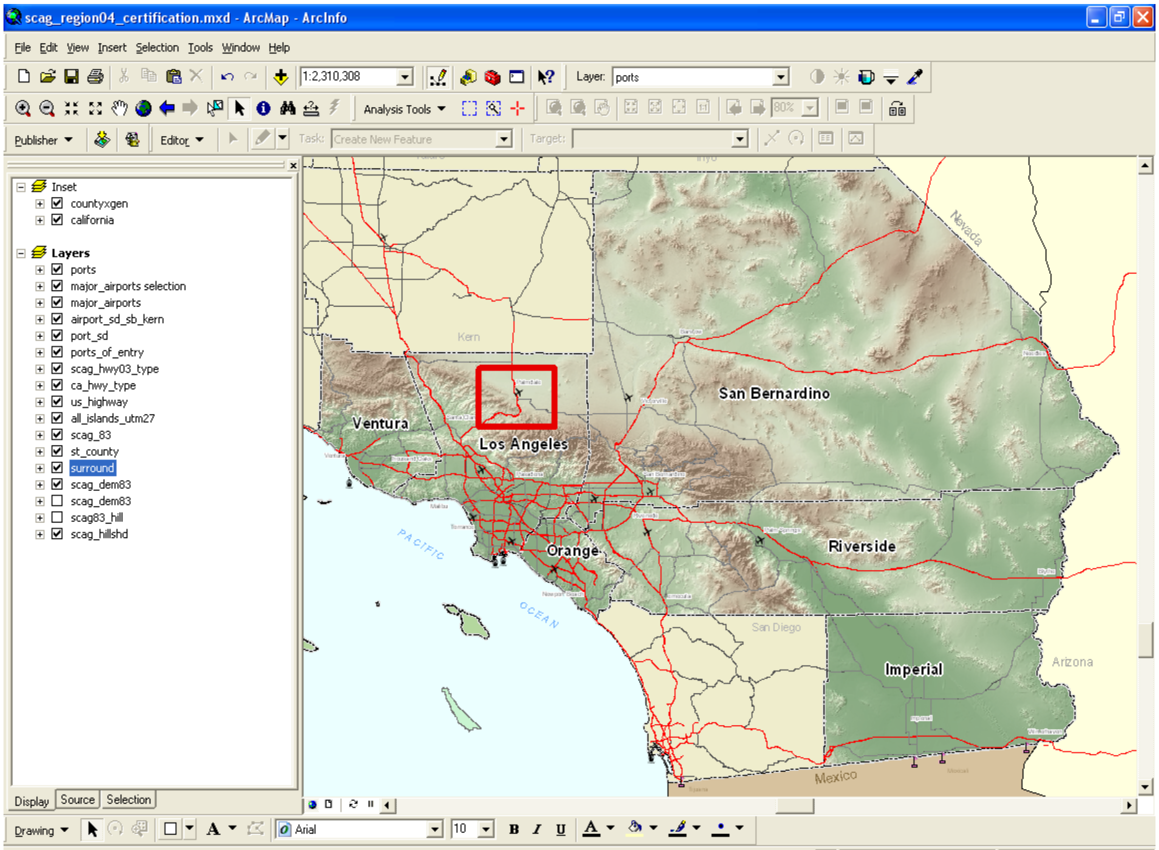

Figure 1. Location of the City of Palmdale in the SCAG Region

Our study focuses on projecting enrollments of "public" senior high school students ranging from 9th grade to 12th grade for analysis purpose during a certain projection period. The total number of enrolled students as of 2005 is assumed approximately 10,000. We expect 5,000 "public" senior high school students be added to the city during the projection period. We also assume that the existing senior high schools can generally accommodate additional 20%-30% of the currently enrolled students. Considering projections of population and additional senior high school students, we assume that one additional senior high school could be added to the City of Palmdale in the target year.

Following the modeling practice of running a facility location model and a spatial interaction model suggested by Ottensmann (2000), our study summarizes the model results of three major modeling processes. They include a facility location, a land suitability analysis, and a spatial interaction model. TAZ is used for our study as unit of analysis.

1) Facility location

There are four existing public senior high schools in the City of Palmdale. Twenty three TAZs are located within the city. Some TAZs are partially located in the city. Since some TAZs are split into two cities or more, total population of the study area should be more than the city level population. Using population size of 2010 as a proxy measure of demands of senior high school enrollments, the preliminary location of a new senior high school is determined at a XY coordinate (397939, 3836564), which minimizes the total (average) distance between demand points and a new school location (See figures 2, 3, 4). The minimized total distance is computed using several initial XY coordinates and an iterative process. The average distance is found to be 1.7 km. The average distance minimized by the facility location model is much shorter than the result of a travel and congestion survey by SCAG in 2001 (See figure 5). According to the SCAG travel and congestion survey, average high school students (7^{th}-12^{th} grade) tend to commute 3.3 km to their high schools with a mode of 1-2 km. Approximately 60% of school trips occur within 2 km of students' home. Distance is Cartesian (straight-line) distance between geocoded home and school locations. Distribution of home to school trips by distance is similar to modified gamma distribution.

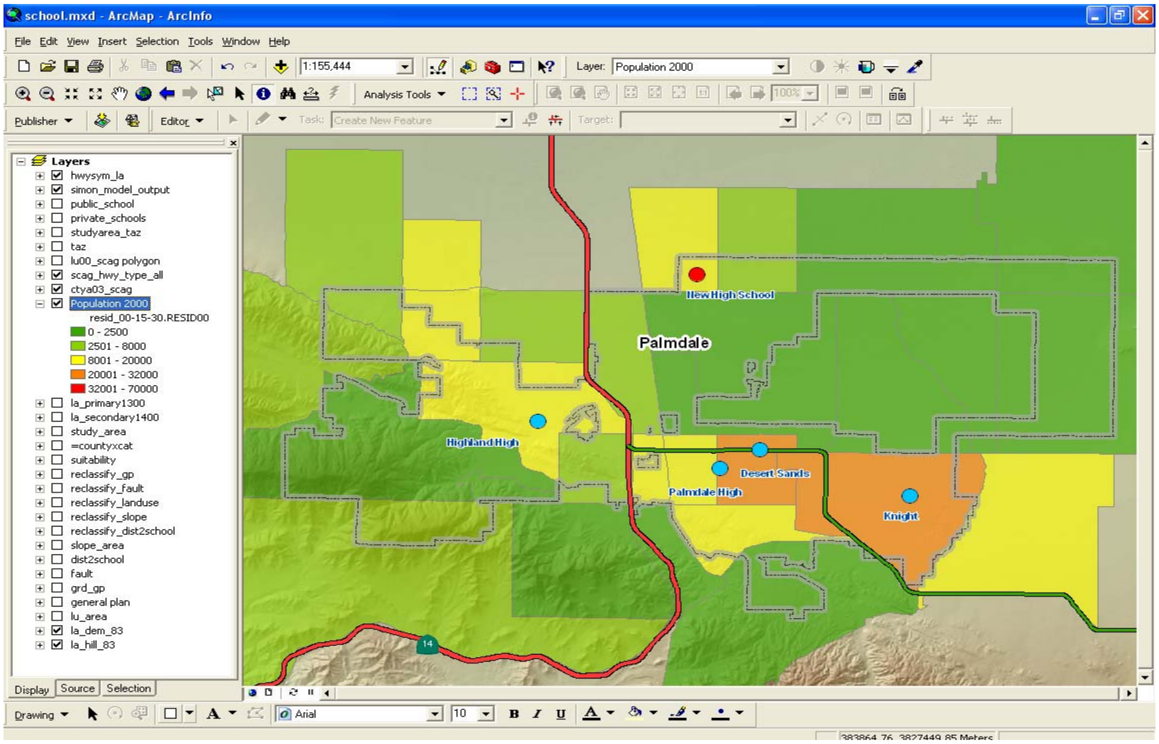

Figure 2. City of Palmdale 2000 residential population by TAZ with location of four existing senior high schools and new senior high school

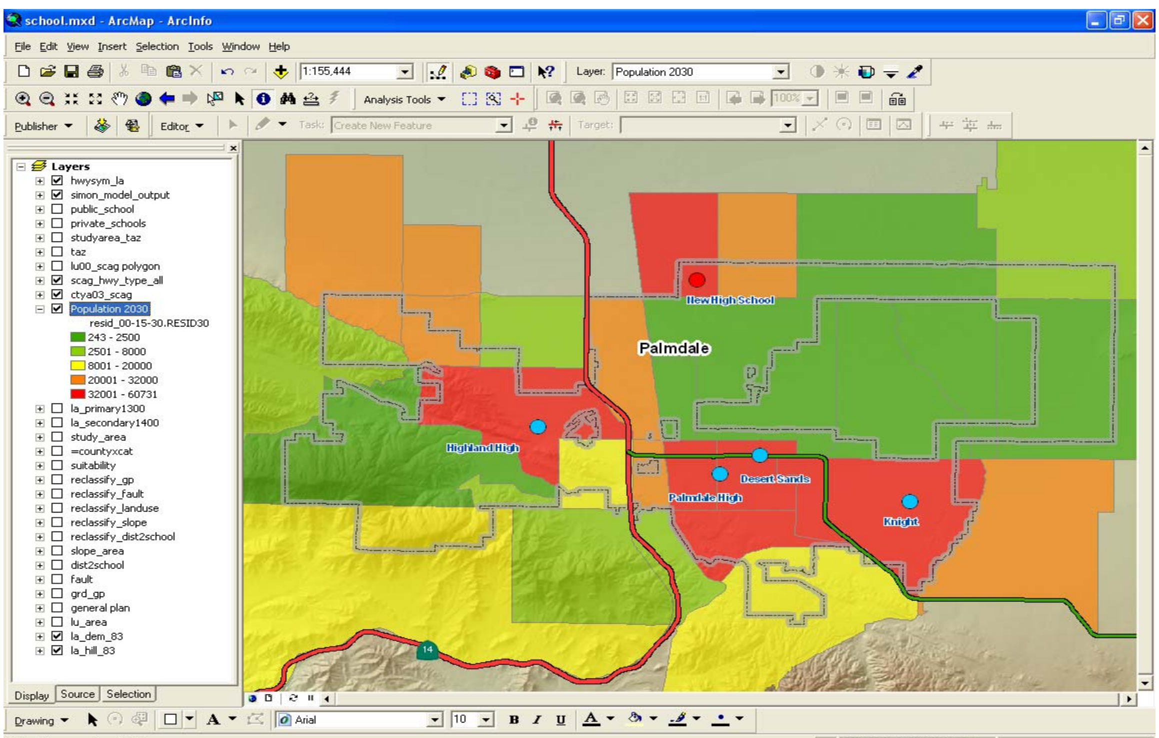

Figure 3. City of Palmdale 2030 residential population by TAZ with location of four existing senior high schools and new senior high school

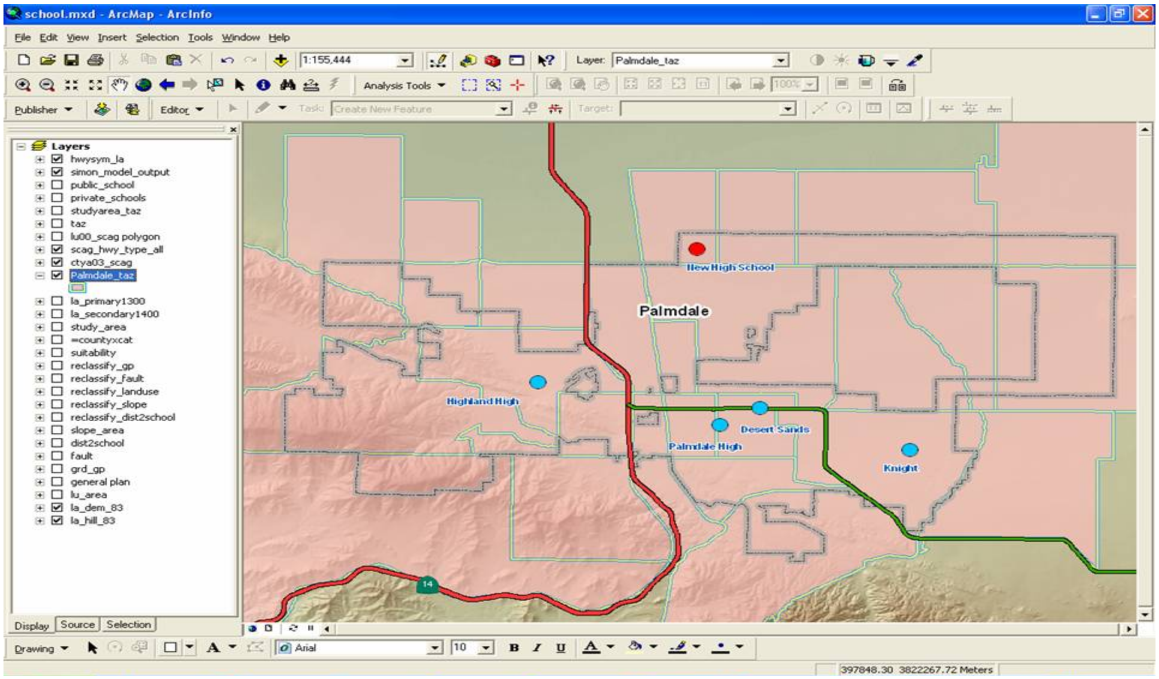

Figure 4. City of Palmdale TAZ boundary with location of four existing senior high schools and new senior high school

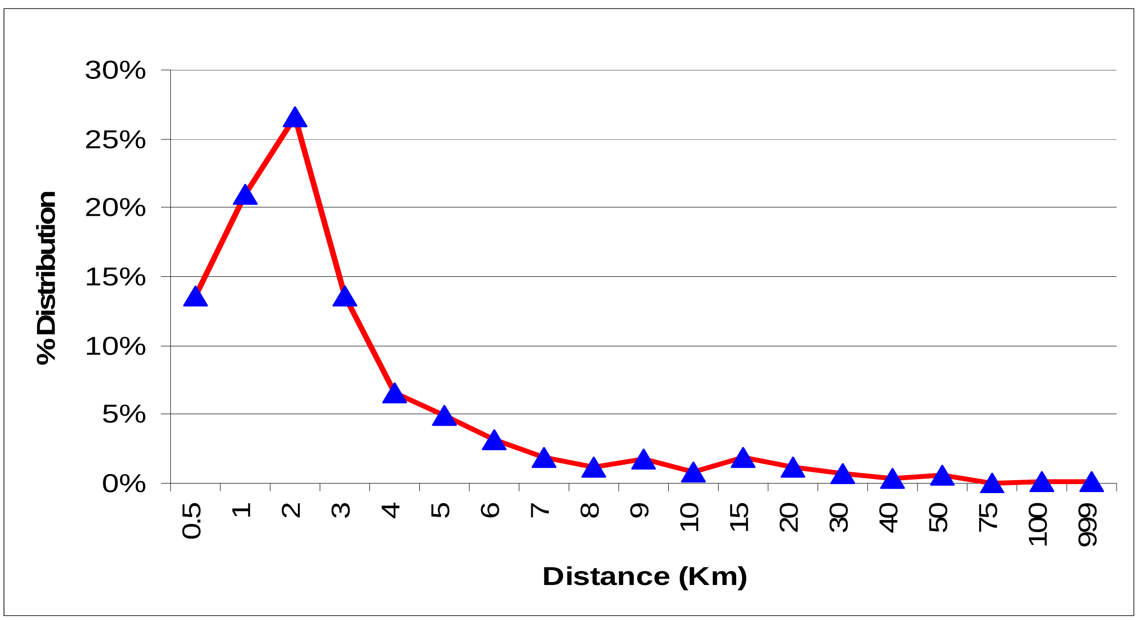

Figure 5. Percent Distribution of Home to School Distance for 7th-12th High School Students
Note: Paul Burke, former SCAG staff (currently LAMTA staff), provided data on percent distribution of home to work distance for enrolled students using 2000 Travel Survey.

The required data set to run a facility location model includes school age population (age 14-17) for each TAZ, current school enrollments for each school, school age population for each TAZ, and XY coordinates of existing schools.

2) Land suitability Analysis

The strength of GIS analysis is to overlay diverse features. Using a weighted rating method, we were able to evaluate a preliminary location for a new school location. In order to apply a weighted rating method, each GIS layer must be measurable and be transformed into a common scale of suitability (usually 1-9). The model returns suitability values on a scale of 1-9 for each cell, where 9 is the best.

To find the best location for a new school by creating a suitability map, two models are being used in our study: 1) surface model[4] creates and analyzes land use and elevation slopes. 2) location model[5] weights and combines multiple raster datasets to help identify the places suitable for schools. The procedure of developing a land suitability model is as follows:

a. Define and select datasets:
   The input datasets in our study are land use, general plan, elevation and slope, fault.

b. Develop Raster datasets:
   Using surface model to derive slope from elevation to raster cells format, converting land use, general plan, and fault (ground water acceleration) features to raster cell format.

c. Reclassify datasets to common scale:
   GIS layers are reclassified to 1 to 10 of a common scale. Within the range 1-10, we give higher values to attributes within each dataset that are more suitable for locating the school:

   - Reclassifying land use: a lower value indicates that a particular land use type is less suitable for building on.
     - Agriculture – 9
     - Vacant – 9
     - Open Space and recreation – 7
     - Residential and Commercial – 6
     - Industrial – 5
     - Transportation – 4
     - Water – 1

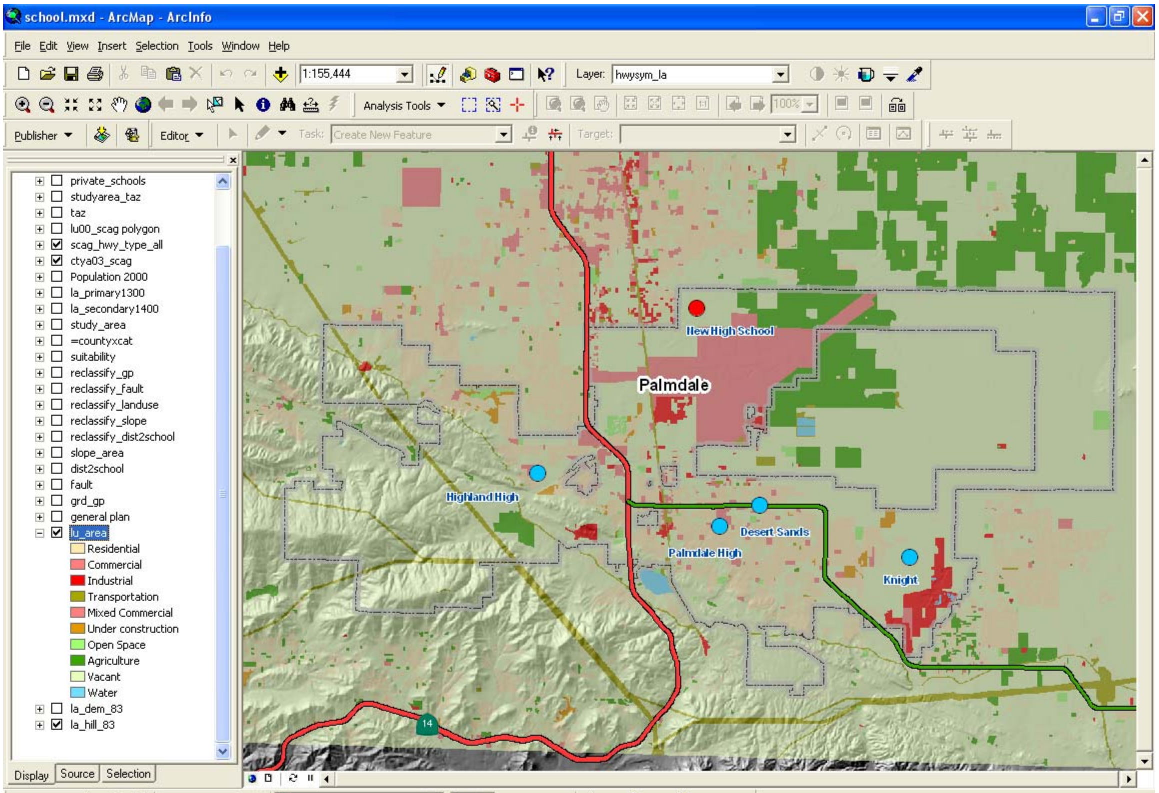

Figure 6. Land use by type

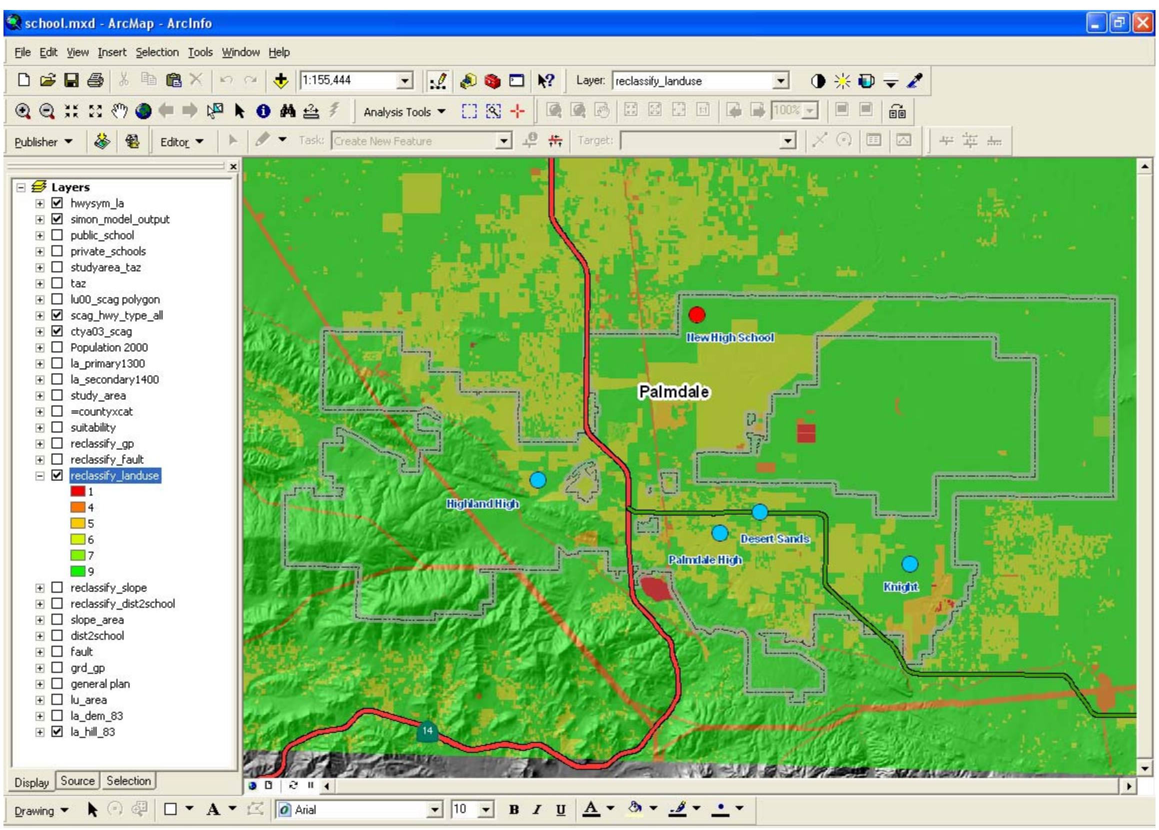

Figure 7. Reclassified land use

- Reclassifying slope: a lower value indicates least suitable for building on (those with the steepest angle of slope).
  - Select 10 classes with equal interval
    - The Lowest Angle of Slope – 10
    - The Steepest Angle of Slope – 1
    - No Data – No data

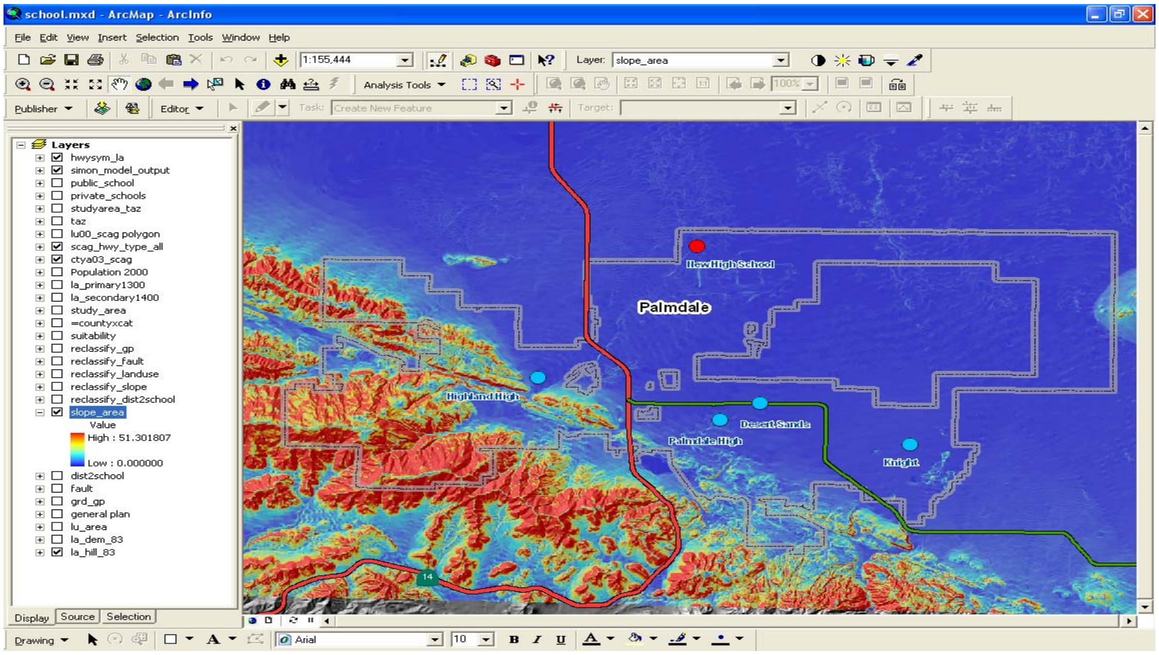

Figure 8. Slope with a raw value

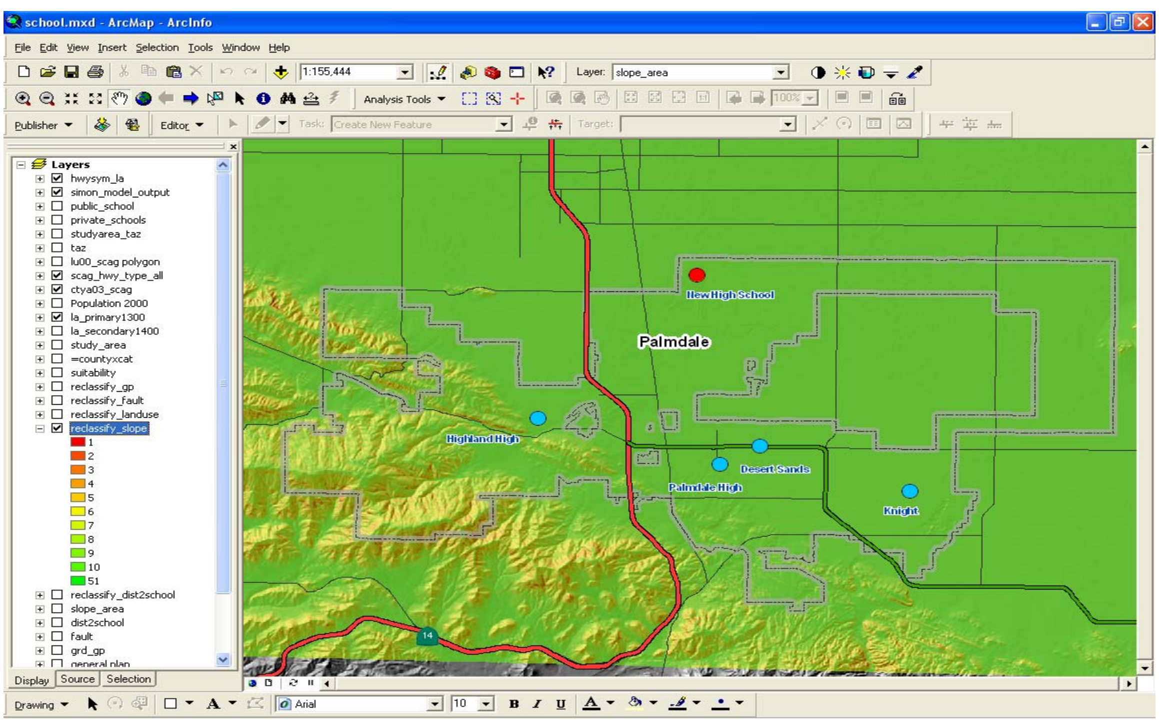

Figure 9. Reclassified slope

- Reclassifying fault: a lower value indicates that a particular land is less suitable for building on.
  - Average 75 – 1
  - Average 65 – 2
  - No data – 10

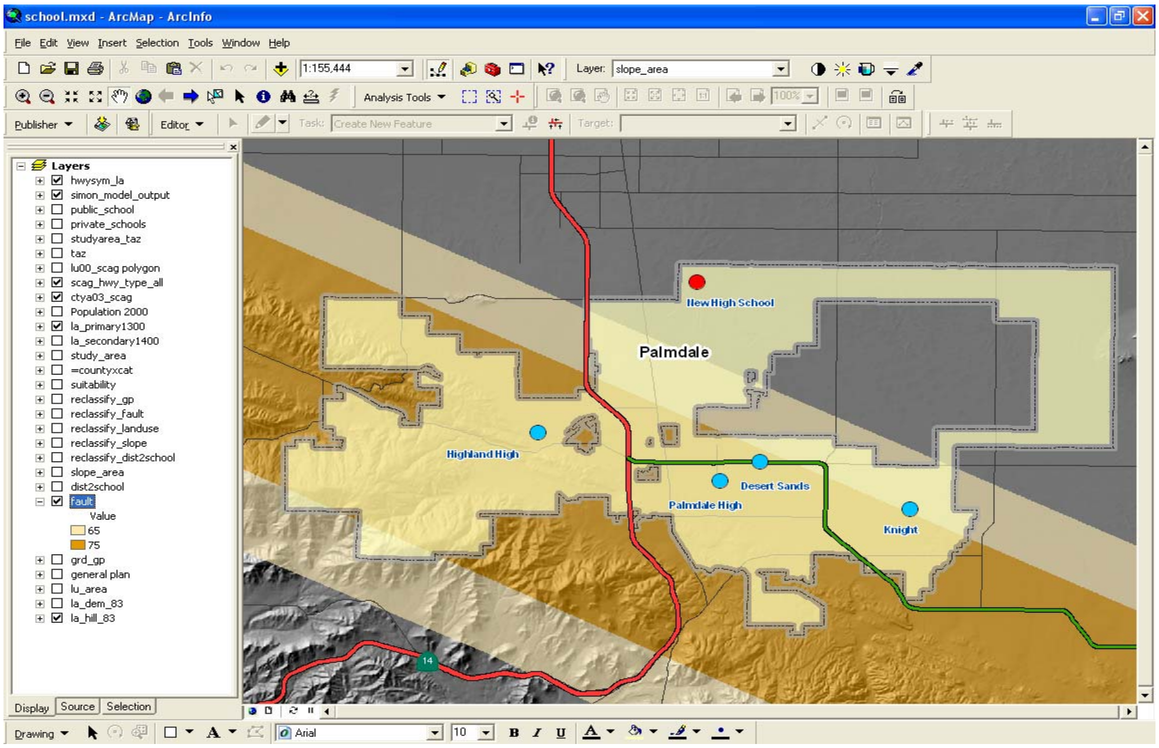

Figure 10. Fault with a raw value

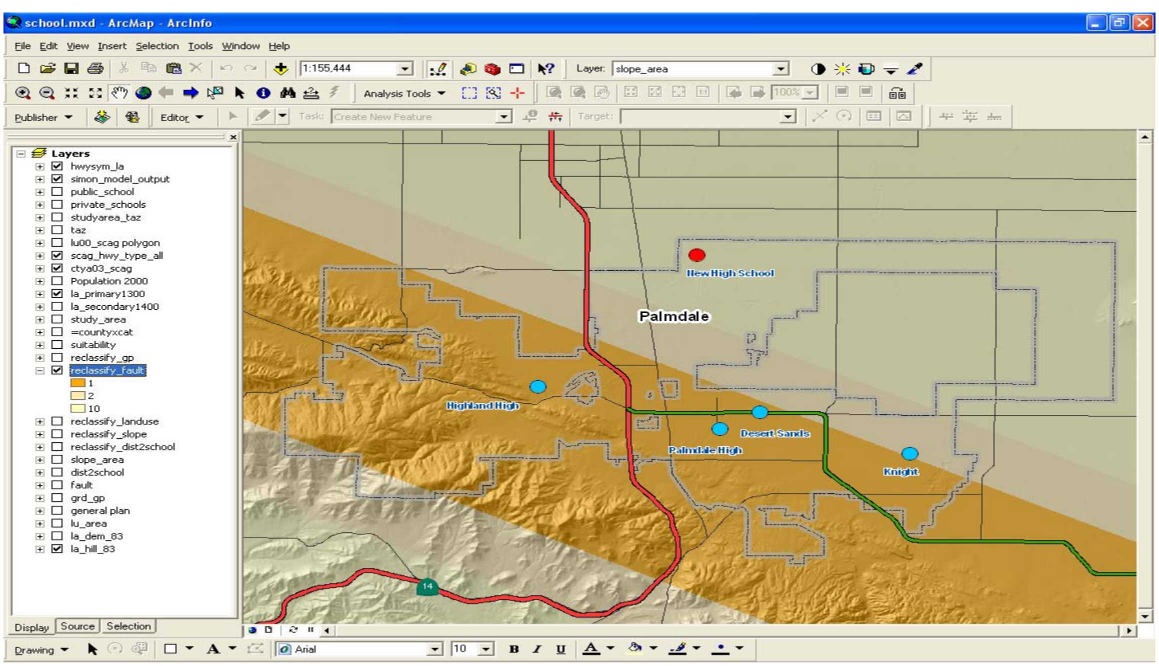

Figure 11. Reclassified fault

- Reclassifying general plan: a lower value indicates that a particular land is less suitable for building on.
  - Commercial – 6
  - Industrial – 5
  - Education – 8
  - Transportation – 4
  - Agriculture – 9
  - Open Non-developable – 1
  - Cemetery – 1
  - Golf – 7
  - Parks – 7
  - Urban Mix – 9
  - Residential – 9

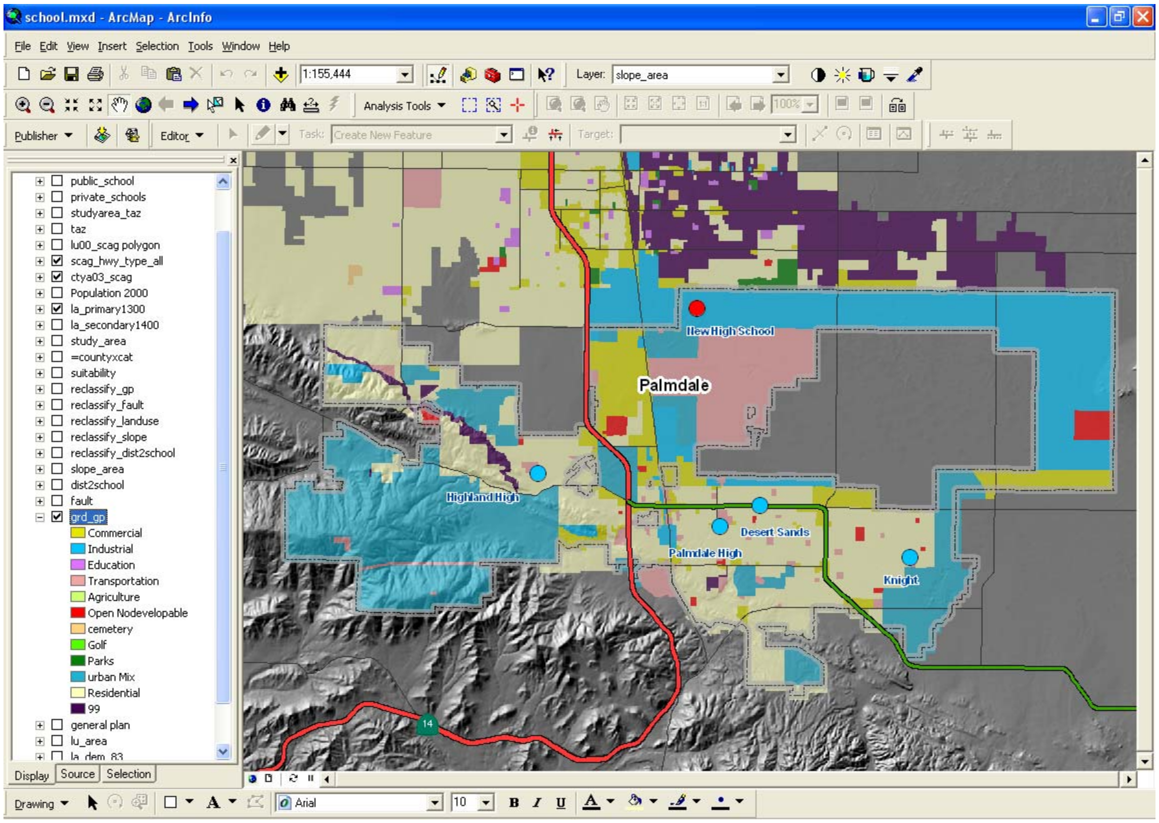

Figure 12. Planned land use by type in the general plan

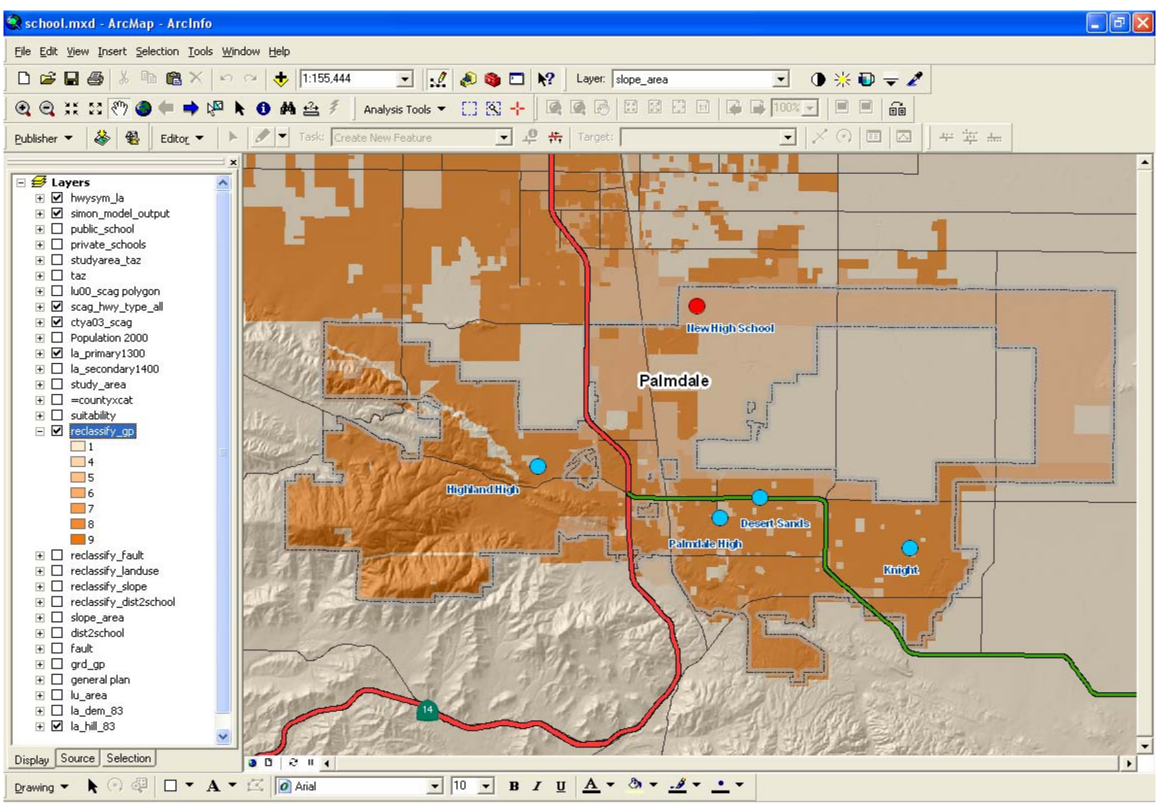

Figure 13. Reclassified planned land use by type in the general plan

d. Weight the datasets

Due to the fact that not all datasets are equally important for the study, we give the datasets the following percent influence (weighting)

- Fault 0.4 (40%)
- General Plan 0.25 (25%)
- Land use 0.20 (20%)
- Slope 0.15 (15%)

e. Combine the datasets to find the most suitable locations:
The reclassified layers are combined in Map Algebra[6] by its weight, expressed as a percentage. The output raster dataset shows how suitable each cell location is for locating the new school. According to criteria setting in the suitability model, higher values indicate sites that are more suitable for school locations.

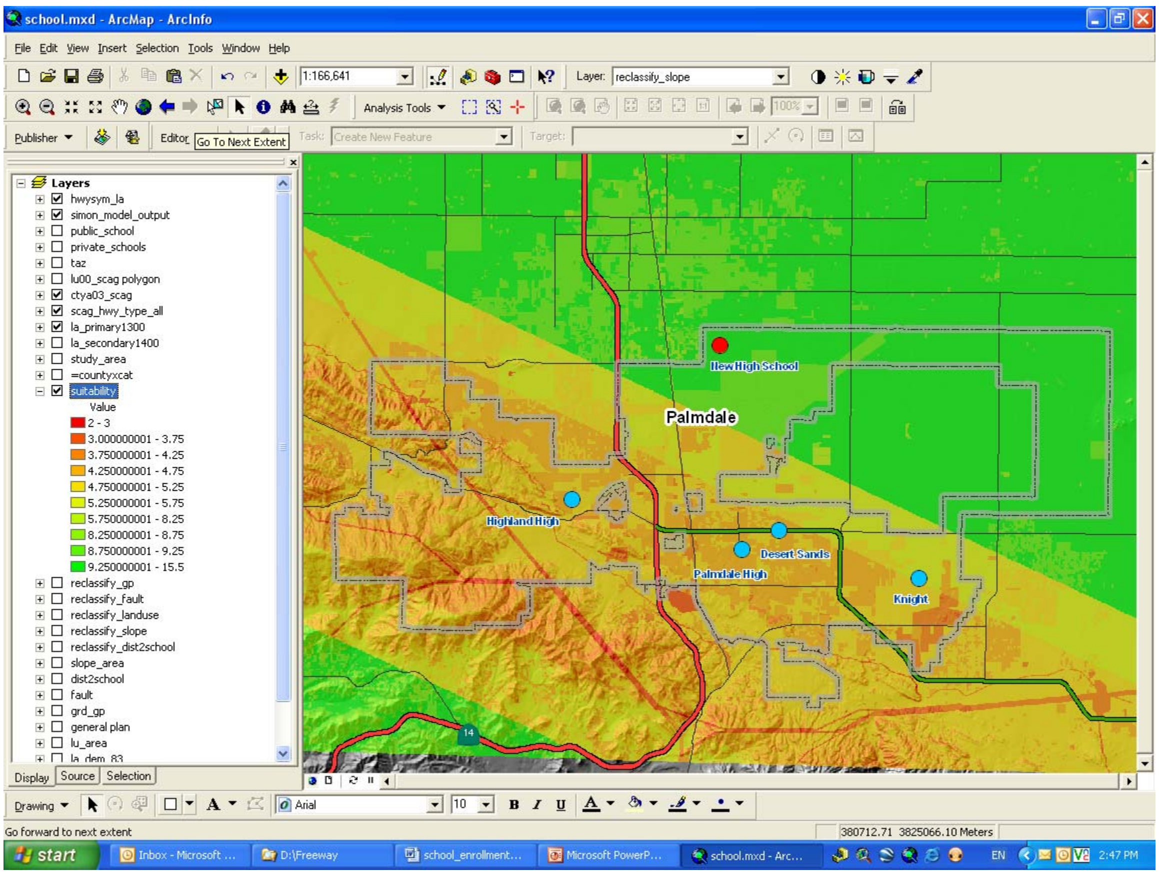

Figure 14. Suitability score

3) Spatial Interaction Model

The initial step of running a spatial interaction model is to estimate two parameters (α and β) to determine the interaction of attractiveness and distance decay function along with a couple of assumptions. Assumptions include: 1) the distance decay function is based on the declining power function. ; 2) The number of classes per school is used for measuring the relative attractiveness of each school. Given the same distance from home to school, the more number of classes per school will have more attractiveness.

Parameters α and β are estimated to be 5.1, 1.2, respectively. A balancing factor *K* is derived as a result of model calibration and determined at 0.058. The maximized log likelihood function is estimated at -12808.6. The calibrated model is represented as follows:

$$D_j = 0.058 \sum_i \left[\frac{O_i A_{ji} f(d_{ij})}{\sum_j A_j^{1.2} f(d_{ij})} \right]$$

Where $f(d_{ij}) = \dfrac{1}{d_{ij}^{5.1}}$

A few evaluation criteria can be used to measure the error in model estimates. They include root mean squared error, mean absolute error, mean absolute percent error (MAPE), mean algebraic percent error (MALPE), etc. (Smith et al, 2001). In our study, RMSE and MAE are estimated to be 63.5 and 53.1, respectively.

4. Results

The existing method generally produces student enrollment projections without consideration of student enrollment capacity of schools and building a new school. As a result, student enrollment projections in the target year could be much higher than the base student enrollments (See table 2). An existing method can not be used for the long term student enrollment projections of small areas.

Table 2. School Enrollment Projections with an Existing Method

School Name	Base Year	Target Year					
		Existing (5,000)		Existing (10,000)		Existing (15,000)	
	Number	Number	%growth	Number	%growth	Number	%growth
Highland High	3597	5406	50%	7215	101%	9023	151%
Palmdale High	3489	5244	50%	6998	101%	8753	151%
Desert Sands Charter	1163	1748	50%	2333	101%	2918	151%
Knight (William J. 'Pete') High	1694	2546	50%	3398	101%	4250	151%
New High							

Note: Existing: Uses each school's share of the city's total high school students as of base year with different student enrollments (5,000, 10,000, 15,000).

Three modeling processes (facility location model, suitability analysis, and spatial interaction model) were used to project high school enrollments, given the assumption of future school enrollment projection at the city level (5,000 additional student enrollments during the projection period). Facility location model produced a preliminary location for one additional new high school. Land suitability analysis was performed using a weighted rating method to evaluate a preliminary location of a new school. Finally, a spatial interaction model was used to produce student enrollment projections with several scenarios of future school enrollments with different assumptions of attractiveness measures (total number of classes per school). The resulted school enrollment projections seem more accurate and reasonable. Spatial interaction model has strength in showing a range of alternatives with different assumptions. Our study developed four alternatives of allocating additional 5,000 student enrollments with following assumptions: 1) no new high school in the target year (alternative 1); 2) increased student capacity of Knight High school and no new high school in the target year (alternative 2); 3) one new high school in the target year (alternative 3); 4) increased student capacity of Knight High school and one new high school in the target year (alternative 4)(See table 3).

There is no change in parameters α and β. A balancing factor K will become 0.049 to allocate additional 5,000 student enrollments in the target year.

Table 3. School Enrollment Projections with Modeling Scenarios

School Name	Target Year							
	Alternative 1		Alternative 2		Alternative 3		Alternative 4	
	Number	% growth	Number	% Growth	Number	% growth	Number	% growth
Highland High	5985	66%	5765	60%	4663	30%	4601	28%
Palmdale High	4957	42%	4479	28%	4104	18%	3811	9%
Desert Sands Charter	1627	40%	1494	28%	1317	13%	1258	8%
Knight (William J. 'Pete') High	2374	40%	3205	89%	2263	34%	2733	61%
New High					2596		2540	

Alternative 1 indicates that Highland High shows more enrollments than those of an existing method, while other three schools show less enrollments than those of an existing method. Highland High seems to be located in an area, which is very accessible from the residential location of students, relative to other schools. Student enrollment projections of alternative one might not be acceptable due to no consideration of school capacity and building a new school. There could be discussion of changing a school schedule from a regular schedule to a year-round schedule.

Alternative 2 is based on realistic assumption of increased student capacity of Knight High school in the near future. Since this high school was established in 2003 and currently occupies only 9^{th}-10^{th} grade students only, the increased capacity reflects the possible increase of enrollments associated with additional 11^{th}-12^{th} grade students. This alternative still does not introduce an assumption of building new high school in the target year. Highland High still shows high level of school enrollments, 60% above that of the base year. This alternative is also hard to be accepted unless there is other assumption of implementing a year-round school schedule.

Alternative 3 is computed using assumptions of one new high school in the target year. The preliminary location of a new high school was identified using a facility location model and confirmed if the site is suitable for a new high school location. The result of alternative 3 shows school enrollment projections for existing and new schools. Knight High does not expect additional enrollments associated with addition of 11^{th}-12^{th} grade in the near future due to a constant enrollment capacity. The result of alternative 3 also shows reasonable school enrollment projections for existing and new schools. In particular, Highland High does not experience a dramatic increase in school enrollments, which was observed from an existing method, alternative 1 and alternative 2.

Alternative 4 is derived using assumptions of increased student capacity of Knight High school and one new high school in the target year. The preliminary location of a new high school was identified using a facility location model and confirmed if the site is suitable for a new high school location.

In summary, alternative 4 produces most reasonable enrollment projections among alternatives, but alternative 3 also can produce acceptable enrollment projections within a framework of limited resources and data availability.

5. Conclusion and Future Research

The purpose of our study is to develop a new approach using GIS and modeling to improve the accuracy of small area school enrollment projections, used as input to running regional transportation model for Southern California. Accurate school enrollment projections would help to increase the credibility of the regional transportation model results.

The new approach utilizes a facility location model, land suitability analysis, a spatial interaction model to develop location of a new school and related school enrollments. GIS techniques including surface model and location model are used in the land suitability analysis.

Our study indicates that modeling processes with reasonable assumptions (e.g., school capacity and a new school) can produce accurate and reasonable student enrollment projections of small areas. The new approach is expected to be used as a planning tool for environmental planners, land use planners, and school district demographers as well as transportation planners, because the new approach has strong planning implication for future accessibility, mobility, air quality, environment, and use of energy (e.g., gasoline).

There is need for future research on several issues to make the proposed modeling process more effective for planning process. First, projections of total population were used for measuring the number of high school age population (14-17 years olds). The projections of reasonable age-specific population would be needed to properly link demographic change and school enrollments.

Second, there was inconsistency of data sets. Geocoded location of schools available from California Department of Education (CA DOE) can not be overlaid with the land use data from SCAG. The reasons include: 1) the problem seems from geocoding of school locations of CA DOE. XY coordinated data from CA DOE contains 3 or 4 decimal points, but need 6 decimal points for more accuracy. CA DOE indicates that it may take time to release such data.; 2) School addresses may include school headquarters, school buildings and campus.; 3) Geocoded school addresses are located at streets, not on campuses. As a result, the number of classes for each school was used as a proxy variable measuring attractiveness of each school.

Third, the modeling process was based on TAZ level allocation with city control. School district level school enrollments control might be better than the city level school enrollments control. School district is a statistical identified boundary processed by the U.S Census Bureau. School district is charged with projecting the future school needs and developing the location strategy and plan for new schools. More communication with school district demographers and planners might be needed to understand the short term forecasts of school enrollments at the small area level.

Fourth and last, the integrated modeling process in a GIS framework would help to develop more accurate and reasonable school enrollment projections in an efficient way. The integrated modeling process is expected to be used as a planning tool for environmental planners, land use planners, and school district demographers as well as transportation planners, because the integrated process has strong planning implication for future accessibility, mobility, air quality, environment, and use of energy (e.g., gasoline). However, developing an integrated modeling process will require a comprehensive analysis of resource needs.

Notes

1. K-12 school enrollment is the total number of K-12 (Kindergarten through 12^{th} grade) students enrolled in all public and private schools located within a TAZ." College/University enrollment is the total number of students enrolled in any public or private post-secondary schools (college or university), that grant an associate degree or higher, located within a TAZ. (SCAG, 2003)
2. Small area projections refer to the growth forecasts done at the Census Tract and Transportation Analysis Zone (CT-TAZ). There are over 8,000 CT-TAZs. (SCAG, 2004). The CT-TAZ level data can be aggregated to the CT, TAZ, city, subregion, county, and regional level.
3. $d_{ij} = \sqrt{(x_i - p_j)^2 + (y_i - q_j)^2}$, where (x_i,y_i) represents the center of demand points, while (p_j, q_j) represents the optimal location.
4. Surface model is surface analysis tools that new information for various locations in a study area can be derived by producing new dataset and identifying patterns in existing surface. Surface Model can generate contours (lines of equal elevation, commonly used in cartography), slope, direction of slope called aspect (measure of the steepness and orientation of the surface), shaded relief called hillshades (measure of the incident light on a surface, commonly used in cartography), and visibility (determines which parts of the surface are visible to an observer).
5. Location model is used to find optimal locations in Suitability model. It includes reclassifying of GIS features dataset, weighting of the features, and combining of the features.
6. Map Algebra is the most powerful feature of ArcGIS Spatial Analyst. By combining its operators and functions, users can perform very sophisticated analysis. (ESRI, White paper)

References

Allen, Eliot. 2001. Index: Software for Community Indicators. Brail, Richard K. and Richard E. Klosterman (eds.), Planning Support Systems. Redlands, California: ESRI Press.

Berke, R. Philippe, David R. Godschalk, and Edward J. Kaiser with Daniel A. Rodriguez. 2005. Urban Land Use Planning Fifth Edition. Illinois: University of Illinois Press.

California Department of Education. 2000. Guide to School Site Analysis and Development 2000 Edition. Sacramento: California Department of Education.

Childs, Colin, and Gary Kabot. Working with ArcGIS Spatial Analyst.

Davis, H. Craig. 1995. Demographic Projection Techniques for Regions and Smaller Areas. Vancouver, Canada: UBC Press.

George, M.V., Stanley K. Smith, David A. Swanson, and Jeff Tayman. 2004. Population Projections. Siegel, Jacob S. and David A. Swanson (eds.). The Methods and Materials of Demography Second Edition. New York: Elsevier Academic Press.

Klosterman, Richard E. 2001. The What-If? Planning Support System. Brail, Richard K. and Richard E. Klosterman (eds.), Planning Support Systems. Redlands, California: ESRI Press.

McCoy, Jill, and Kevin Johnston. 2001. Using ArcGIS Spatial Analyst. Redlands, California: ESRI Press.

Nelson. Arthur C. 2004. Planner's Estimating Guide Projecting Land-Use and facility Needs. Chicago: Planners Press, American Planning Association.

Ottensmann, John R. 1985. Basic Microcomputer Programs for Urban Analysis and Planning. New York: Chapman and Hall.

Ottensmann, John R. 2000. Basic Microcomputer Programs for Urban Analysis and Planning. New York: Chapman and Hall.

Plane, David A. and Peter A. Rogerson.1994. The Geographical Analysis of Population with Application to Planning and Business. New York: John Wiley & Sons, Inc.

Randolph, John. 2004. Environmental Land Use Planning and Management. Washington, D.C.: Island Press.

Southern California Association of Governments. 2003. Year 2000 Model Validation & Summary: Regional Transportation Model.

Southern California Association of Governments. 2004. 2004 Regional Transportation Plan: Technical Appendix A. Growth Forecast.

Siegel, Jacob S. 2002. Applied Demography: Applications to Business, Government, Law, and Public Policy. San Diego: Academic Press.

Smith, Stanley K., Jeff Tayman, David A. Swanson. 2001. State and Local Population Projections: Methodology and Analysis. New York: Kluwer Academic/Plenum Plenum Publishers.

Yoon D., S. Yoon. 2004. Urban Modeling: Techniques and Application. Seoul, Korea: HongMoonSa.